Mixed Conics worksheet #1-10

parabola (one squared term)

(1.) $x^2-4y-6x+9=0$

$x^2-6x+\_\_ = 4y-9+\_\_$

$x^2-6x+\underline{9} = 4y-9+\underline{9}$

$\boxed{(x-3)^2 = 4y}$

positive, opens right

$4p=4$ ← focal diameter

$p=1$ ← distance to focus & directrix

$(h,k)=(3,0)$ vertex

Circle (add, equal coeff for x^2 + y^2)

(2.) $x^2-8x+y^2+6y+24=0$

$x^2-8x+\_\_+y^2+6y+\_\_ = -24+\_\_+\_\_$

$x^2-8x+\underline{16}+y^2+6y+\underline{9} = -24+\underline{16}+\underline{9}$

$\boxed{(x-4)^2+(y+3)^2 = 1}$ ← r^2

Center $(h,k)=(4,-3)$

radius $=\sqrt{1}$

$r=1$

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hyperbola (subtract, coefficients not equal)

(3.) $x^2-3y^2+2x-24y-41=0$

$x^2+2x+\_\_ -3y^2-24y = 41$ (*) factor -3 carefully from "y" terms

$x^2+2x+\_\_ -3(y^2+8y+\_\_) = 41+\_\_+\_\_$ careful

$x^2+2x+\underline{1} -3(y^2+8y+\underline{16}) = 41+\underline{1}+\underline{-48}$ ← $-3(16)=-48$

$42+-48$

$$\frac{(x+1)^2}{-6} - \frac{3(y+4)^2}{-6} = \frac{-6}{-6}$$

this is now the positive term

$$\frac{(x+1)^2}{-6} + \frac{(y+4)^2}{2} = 1$$

or $\boxed{\dfrac{(y+\overset{k}{4})^2}{2} - \dfrac{(x+\overset{h}{1})^2}{6} = 1}$ vertical orientation since y^2 is positive

a^2 positive term if hyperbola (might be larger or smaller than b^2)

b^2

(*) Only ellipse has the largest a^2 value

use a + b to find box + asymptotes

$a^2=2$

$a=\sqrt{2}$ up/down

≈ 1.4

$b^2=6$ keep exact value

$b=\sqrt{6}$ left/right

≈ 2.4 for graphing purposes

Center $(h,k)=(-1,-4)$

coefficients not equal → $\boxed{\text{Ellipse}}$

add

(4.) $9x^2 + 25y^2 - 54x - 50y - 119 = 0$

careful!

$9(9) \qquad 25(1)$

$9x^2 - 54x + 25y^2 - 50y = 119$

$9(x^2 - 6x + \underline{9}) + 25(y^2 - 2y + \underline{1}) = 119 + \underline{81} + \underline{25}$

$\dfrac{9(x-3)^2}{225} + \dfrac{25(y-1)^2}{225} = \dfrac{225}{225}$

$$\boxed{\frac{(x-3)^2}{25} + \frac{(y-1)^2}{9} = 1}$$

largest $= a^2$ for ellipse

b^2

center $= (3, 1)$
$\quad\quad\quad\;\; h \; k$

$a = 5$ (left + right) $\qquad b = 3$ (up + down)

$\boxed{\text{parabola}}$

(5.) $x^2 = y + 8x - 16$

$x^2 - 8x + \_\_ = y - 16 + \_\_$

$x^2 - 8x + \underline{16} = y - 16 + \underline{16}$

$(x-4)^2 = y$

$4p = 1$ ← focal diameter

(h, k)

vertex $= (4, 0) \qquad p = \frac{1}{4}$

(6.) $x^2 - 4x - y^2 - 5 - 4y = 0$

careful!! (factor -1)

$\boxed{\text{hyperbola}}$

negative squared term

$x^2 - 4x - y^2 - 4y = 5$

$x^2 - 4x + \_\_ - (y^2 + 4y + \_\_) = 5 + \_\_ + \_\_$

$x^2 - 4x + \underline{4} - (y^2 + 4y + \underline{4}) = 5 + 4 - 4$

careful! $-1(4) = -4$

$(x-2)^2 - (y+2)^2 = 5$

$$\boxed{\frac{(x-2)^2}{a^2\;5} - \frac{(y+2)^2}{5\;b^2} = 1}$$

horizontal orientation

use $a + b$ to find box and asymptotes

positive term $= a^2$

center: $(h, k) = (2, -2)$

$a = \sqrt{5}$ (left + right) $\qquad b = \sqrt{5}$ exact value (up ...) ≈ 2.1 for graphing purposes

$\boxed{\text{ellipse}}$

add

(7.) $5x^2 + 2y^2 - 40x - 20y + 110 = 0$

$5x^2 - 40x + 2y^2 - 20y = -110$

$5(x^2 - 8x + \_\_) + 2(y^2 - 10y + \_\_) = -110 + \_\_ + \_\_$

$5(x^2 - 8x + \underline{16}) + 2(y^2 - 10y + 25) = -110 + \underline{80} + \underline{50}$

careful! $5(16) = 80$
$2(25) = 50$

$$\frac{5(x-4)^2}{20} + \frac{2(y-5)^2}{20} = \frac{20}{20}$$

$$\boxed{\frac{(x-4)^2}{\underset{b^2}{4}} + \frac{(y-5)^2}{\underset{a^2}{10}} = 1}$$

center $(h,k) = (4,5)$

largest value is a^2 for ellipse

$b = 2$ (left + right)

$a = \sqrt{10}$ (up + down) ≈ 3.2 for graphing purposes

(8.) $x^2 - 8x + 11 = -y^2$

$x^2 - 8x + y^2 = -11$

$\boxed{\text{Circle}}$

equal coefficients + positive squared terms

$x^2 - 8x + \_\_ + y^2 = -11 + \_\_$

$x^2 - 8x + \underline{16} + y^2 = -11 + \underline{16}$

$\boxed{(x-4)^2 + y^2 = \underset{r^2}{5}}$

center $(h,k) = (4,0)$

$r = \sqrt{5}$
≈ 2.1

hyperbola

pos, neg

(9.) $8y^2 - 9x^2 - 16y + 36x - 100 = 0$

$8y^2 - 16y - 9x^2 + 36x = 100$

careful! $8(y^2 - 2y + \_\_) - 9(x^2 - 4x + \_\_) = 100 + \_\_ + \_\_$ careful!

$8(y^2 - 2y + \underline{1}) - 9(x^2 - 4x + \underline{4}) = 100 + 8 - 36$

8(1), -9(4)

$$\frac{8(y-1)^2}{72} - \frac{9(x-2)^2}{72} = \frac{72}{72}$$

vertical $\boxed{\dfrac{(y-1)^2}{9} - \dfrac{(x-2)^2}{8} = 1}$

a^2 is with the positive term, b^2

always with x, always with y

center: $(h,k) = (2,1)$

use a & b to find box & asymptotes

$a = 9$ (up & down) $b = \sqrt{8}$ (left & right)

≈ 2.8 for graphing purposes

parabola

one squared term

(10.) $4y^2 + 4y + 8x = 15$

$4(y^2 + y) = -8x + 15$

$4(y^2 + y + \_\_) = -8x + 15 + \_\_$ careful! $4(\frac{1}{4}) = 1$

$4(y^2 + y + \frac{1}{4}) = -8x + 15 + 1$

$4(y + \frac{1}{2})^2 = -8x + 16$

$$\frac{4(y + \frac{1}{2})^2}{4} = \frac{-8(x-2)}{4}$$

$\boxed{(y + \frac{1}{2})^2 = -2(x-2)}$ Vertex $(h,k) = (2, -\frac{1}{2})$

negative opens to left

$4p = -2$ ← focal diameter $= |-2| = 2$

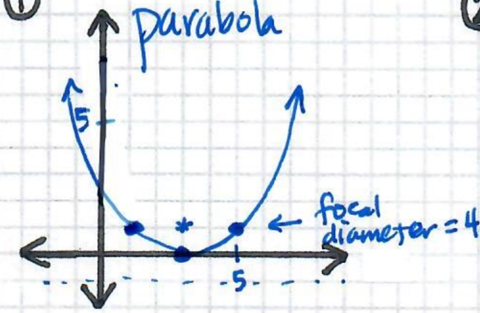
$p = -\frac{1}{2}$

distance to focus & directrix is $|-\frac{1}{2}| = \frac{1}{2}$

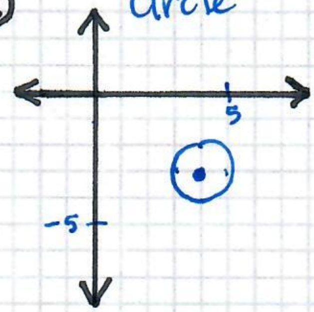
Mixed Conics

Name: Key Per:

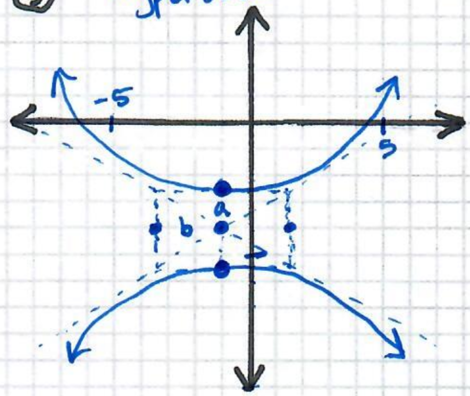
1. parabola



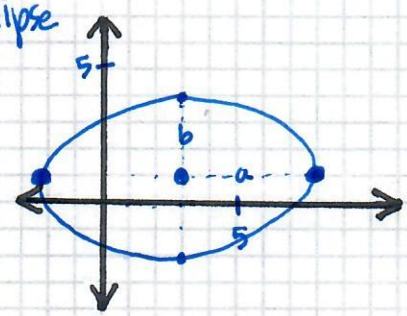
2. circle



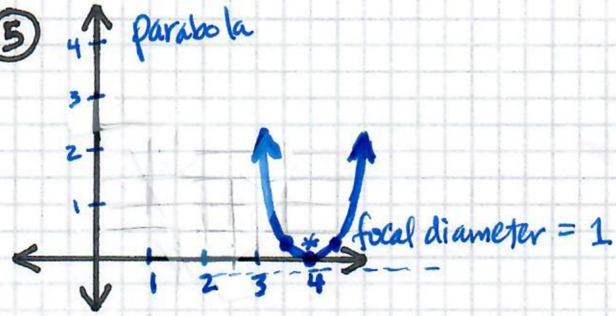
3. hyperbola



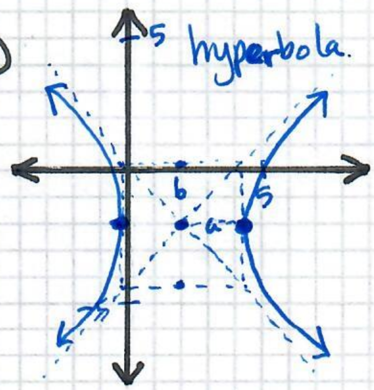
4. ellipse



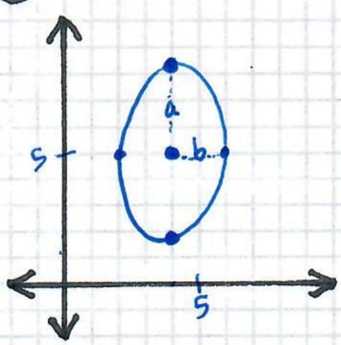
5. parabola



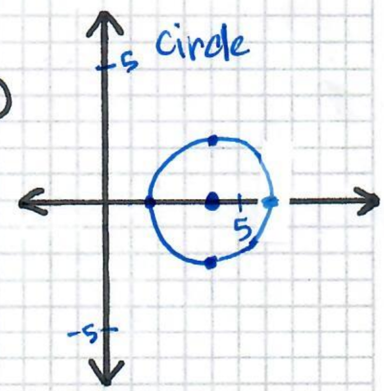
6. hyperbola



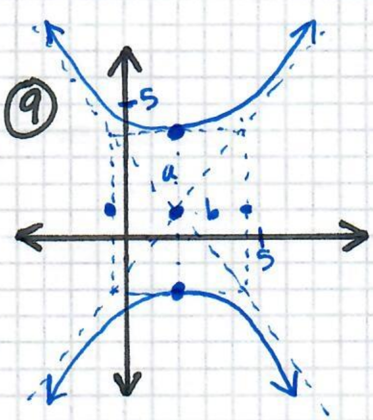
7. ellipse



8. circle



9. hyperbola



10. parabola